

# INCOME PROTECTION – TECHNICAL REPORT

AUGUST 1997

Joseph A. Atwood, Alan E. Baquet, and Myles J. Watts

This report presents the principles used in developing the income protection (IP) insurance policy as well as the procedures used to estimate the corresponding premia. The changes that have been made in the rating procedures since the last technical report dated February 16, 1996 are contained in this report. A general description of the rating procedure is presented followed by a more technical discussion.

## The Income Protection Policy

The income protection (IP) policy insures producers against low-income events due to low prices and/or low yields. In contrast to traditional yield based insurance policies the IP policy will not provide indemnities if yields are low but prices are sufficiently high so that revenue exceeds the payment trigger levels. Conversely, the income protection policy will pay indemnities at higher yield levels if prices are sufficiently low so that revenue falls below the payment trigger levels. Payment scenarios for IP and traditional yield insurance are portrayed in Figure 1. In both cases the assumed APH yield is 100 units with an expected price of \$2.00 per unit of output. Both the yield and IP payment triggers are assumed to be 75% of the APH yield times the \$2.00 price.

The current multiple peril yield based insurance incurs indemnities whenever yields fall below 75%, regardless of the price received for the quantity produced. If the actual price received exceeds the expected price of \$2.00, producer revenue may exceed \$150 (75% of the expected gross revenue) and they would also receive additional yield insurance indemnities. These events are graphically portrayed by the area above and to the right of the curve  $P \cdot Y=150$  and to the left of the line  $Y=75$ .

If producers are concerned with low revenue events rather than low yields, per se, they would be more concerned with events below the curve  $P \cdot Y=150$ . A producer purchasing IP revenue insurance would be giving up payments in the area above the curve  $P \cdot Y=150$  and to the left of  $Y=75$  in return for receiving additional payments for events below the curve  $P \cdot Y=150$  but to the right of  $Y=75$ .

The payoff distribution, and hence the associated premia, will depend upon the position and shape of the joint price-yield probability density function lying over the two dimensional space portrayed in Figure 1. It is not clear, a priori, whether yield or revenue based insurance premia will be more expensive. It is likely that revenue-based premia will be higher in some situations and lower in others.

The remainder of this report describes the Income Protection (IP) insurance product in more detail. We first present an overview of the basic product and then proceed to a more technical discussion of the procedures used to develop the IP rates.

## ■ AN OVERVIEW OF THE IP PRODUCT

An example situation is presented in the following paragraphs. Assume that we have two producers from a hypothetical wheat-producing county in the Midwest. Each producer's farm is assumed to exhibit average productivity and each producer's yields are assumed to equal the county-adjusted-regional yield (CAR), to be described in more detail later. Table 1 lists the hypothetical yields for each producer as well as the CAR yield for their county. As with the APH yield based insurance product, each producer is required to submit from four to ten years of yield history to purchase IP insurance. Producer 1 reports four years of yield data while producer 2 reports ten years. The first producer's four year APH yield is 36.55 bushels per acre. The second producer's ten year APH yield is 32.97 bushels. Assuming a price of \$4.00 per bushel the first producer can select an indemnity trigger between \$73.10 ( $.5 * \$4.0 * 36.55$ ) and \$109.65 ( $.75 * \$4.0 * 36.55$ ). Producer two can select an indemnity trigger between \$65.94 ( $.5 * \$4.0 * 32.97$ ) and \$98.91 ( $.75 * \$4.0 * 32.97$ ). Note that each producer's yields are equal to the CAR yield for the years they reported. By assumption each farm's yields are equal to the CAR yield. Note also that the APH yields of the two producers are 3.58 bushels different resulting in the two producers being eligible for different indemnity triggers although the farms are assumed to be identical in productivity and variability.

The IP product differs from the traditional APH product in that the years for which the yields are reported influence the producer's premia. To see the reason for this examine Figure 2 in which the long term CAR yields (50 years) are shown as well as the estimated long term trend of the CAR yields. In this county two of the three worst yield events in the past 50 years occurred in the ten year period for which producer two reported yields. As can be seen from Table 1, the average CAR yield for those years is 32.97 bushels per acre. The average CAR

yields over the past four years are higher at 36.55 bushels per acre. If the long-term data is used to 'forecast' the region's expected yield; the expected yield for 1997 is 35.5 bushels per acre. For both the APH and IP products the indemnity trigger levels at which the producer receives an indemnity are based off the APH yield of 36.55 for producer one and 32.97 for producer two. If indeed each producer is expected to maintain yields equal to the county, their actual expected yield for 1997 should be 35.5 bushels per acre and their expected revenue is \$142 per acre. In this case the first producer's 75% indemnity trigger of \$109.65 is 77% of the actual expected revenue amount \$142. Producer two's 75% indemnity trigger of \$98.91 is only 70% of the actual expected revenue. Under the current APH rules, producer two is not allowed to purchase insurance coverage equivalent to 75% of expected revenue. Alternatively, producer one is able to purchase coverage in excess of 75% of expected revenue. This is because the most recent four CAR yields were slightly above average with respect to long term expected yields.

The IP rating procedure accounts for any potential bias in APH yields when rates are developed. The exact procedures are described in more detail in the following section. The procedure contrasts the producer's yields to the CAR yields for the same years. If the producer's average APH yield is X units above the average CAR yields for the same years, the producer's expected yield is set at X units above the projected 1997 CAR yield for rate making purposes. Procedurally this is accomplished by calculating the average of the CAR yields for the years reported by the producer. For the example the CAR average yields are the same as the APH yields for each producer giving an average deviation of zero from the region's expected yield.

Given each producer's APH average yield as well as the CAR average yield for the corresponding years, the appropriate IP insurance rates can be obtained from Table 2. To obtain a rate quote for producer one we look under the farm yield columns until we find the row in which the APH yield of 36 falls (yield APH's are rounded down). We then look across the CAR

yield row until we find the column that contains the CAR average yield level of 36. At the intersection of the row and column the rate is 160 or .160. The producer's 75% premium before subsidy is  $.160 * \$109.65$  or \$17.54 per acre. The same procedure is followed for producer two giving a rate of .102 and a per acre premia of \$10.09. The rate charged producer two is less than that charged producer one since producer two is effectively only able to purchase insurance at a level equal to 70% of the farm's expected revenue.

The preceding paragraphs have presented the basic procedures required to obtain an IP premium quote. The following sections of the paper discuss the procedures and data used in estimating rates such as those contained in Table 2. The reader is referred to appendices for a more detailed discussion of the data.

## ■ THE IP RATING PROCEDURES

### ■ The Data

Five types of data are used in the IP rating process. The data types (and notation) are defined as follows:

$$\text{Regional Yield Data} \quad \equiv \quad R_t \quad t = 1, 2, \dots, T_R$$

$$\text{County Yield Data} \quad \equiv \quad C_t \quad t = 1, 2, \dots, T_C$$

$$\text{Pooled FARM Yield Data} \quad \equiv \quad y_t^f \quad t = 1, 2, \dots, T_f \quad f = 1, 2, \dots, F$$

$$\text{Price Data} \quad \equiv \quad P_t^1 / P_t^0 \quad t = 1, 2, \dots, T_p$$

$$\text{Insuree's Yield Data} \quad \equiv \quad y_t \quad t = 1, 2, \dots, T_I$$

In the above expressions, we have  $T_R$  years of regional data,  $T_C$  years of NASS county data,  $T_f$  years of APH data for farm  $f$ ,  $T_p$  years of futures price data, and  $T_I$  years of yield data for the insuree. The regional data are the acre-weighted averages of the NASS county yields for all counties in the Federal Crop Insurance Corporation specified risk-rating regions. The county yields are NASS county yields per planted acre. The pooled FARM data consists of the most recent APH yield data reported by farmers and recorded in the FCIC yield history files. For rate estimation purposes, data is only used from farms that report six or more years of actual yields. The price data are futures price data (See Appendix B). The Insuree's data is the yield data provided by the prospective buyer of insurance.

## ■ STATISTICAL ANALYSIS

### ■ ESTIMATING THE REGIONAL YIELD TREND

The first step in the process is to estimate the regional yield trend as:

$$R_t = a_1^R + g(t) + e_t^R \quad (1)$$

In expression (1),  $a_1^R$  is the regional yield intercept,  $g(t)$  is the region's estimated yield trend over time, and  $e_t^R$  is the regional residual yield variation. The form of  $g(t)$  may vary by region. Appendix A contains a more detailed discussion of the potential functional forms used to estimate the yield trend. It should be emphasized that the same yield trend is enforced upon all counties in the risk-rating region. Estimating  $g(t)$  for each county would allow statistical anomalies to generate different yield predictions and possibly widely different rates within an area that FCIC has determined has a similar yield productivity and risk. We exogenously impose the restriction that all counties in the same area possess the same yield trend,  $g(t)$ . Examples are plotted in Figures 2 and Figures 3.

Figure 3 plots the actual and predicted regional wheat yield for an area in southern Kansas. The planted yield per acre in Kansas is estimated to have a nonlinear trend. The functional form is described in Appendix A (expression (A3)). As previously discussed, Figure 2 plots the actual and predicted wheat yield for a region in central Montana. A linear trend could not be rejected for central Montana (expression (A2) in APENDIX A).

The above expression is estimated using least squares regression. Different procedures, suggested by other researchers, are POTENTIALLY more robust to statistical outliers. These procedures may be considered for future use after statisticians have had sufficient time to

examine the procedure's small sample properties. At the current time least squares procedures are well known and are used in a wide variety of statistical and econometric applications.

## ■ CHECKING FOR REGIONAL YIELD HETEROSKEDASTICITY

The next step in the process is to perform a statistical check on whether the variability of regional yields has changed across time using generalized least squares (GLS).

There are a number of well-known GLS procedures that can be used. One procedure examined was the procedure published by GLEJSER. The coefficients of the following equation are estimated and used to predict:

$$|\hat{e}_t^R| = b_1^R + b_2^R t \quad (2)$$

In expression (2), the term  $|\hat{e}_t^R|$  refers to the PREDICTED level of the absolute regional yield error. The predicted values of the absolute yield errors are used to scale the original yield errors from equation (1) to 1997 units as:

$$e_{t,97}^R = (b_1^R + b_2^R \cdot 1997) / (b_1^R + b_2^R t) \quad (2a)$$

In expression (2a) the term  $e_{t,97}^R$  refers to the regional error from year  $t$  that has been scaled to 1997 units. The scaling process involves the heteroskedastic correction made by dividing by the forecasted value from expression (2) and then rescaling all the corrected errors by the predicted scaling factor for 1997. When scaling we also restrict the process so that no rescaled error can fall below (above) the most negative (positive) unscaled error in the original set of errors from (2). Figure 4 portrays the effect of scaling a set of CAR wheat residuals in Montana using the



GLEJSER procedure. (The actual heteroskedastic correction procedure used will vary depending upon the characteristics of the particular region's data.)

## ■ RECENTERING THE COUNTY DATA AND CONSTRUCTING COUNTY ADJUSTED REGIONAL (CAR) YIELDS

After estimating the regional trend, a county specific intercept is estimated to account for county specific productivity differences. Using NASS data, for any years available, the county specific intercept is estimated from:

$$C_t = a_1^C + g(t) + u_t^C \quad (3)$$

In expression (3),  $g(t)$  is the function estimated for the region in equation (1). If  $g(t)$  is linear, this estimation can be accomplished by using a restricted least squares statistical package. If  $g(t)$  is nonlinear,  $a_1^C$  can be estimated as:

$$a_1^C = \frac{1}{T_C} \sum_{t=1}^{T_C} (C_t - g(t)) \quad (3a)$$

Prior to the 1998 crop year, the  $u_t^C$  county level residuals were used to estimate rates. As the program was expanded, however, it was discovered that a complete set of NASS county data was not available for all years, practices, and counties in many of the risk rating regions. This was sometimes due to NASS's privacy rules concerning survey responses as well as other reasons. In any case, this made using the previous rating procedure infeasible in some counties. To maintain a consistent rating process across regions, it was decided that County Adjusted

Regional (CAR) yields would be calculated for each county in a risk rating region for which regional yields could be constructed. The CAR yield series for each county is constructed as follows:

$$R_t^C = a_1^C + g(t) + e_t^R \quad (4)$$

where  $R_t^C$  is the County Adjusted Regional (CAR) yield,  $a_1^C$  and  $g(t)$  are from expression (3), and  $e_t^R$  are the regional residuals from expression (1). Using  $R_t^C$ ,  $g(t)$ , and  $e_t^R$  from (4), the county specific intercept  $a_1^C$  is bootstrapped to account for the uncertainty in the estimated parameter (see Appendix C).

## ■ USING THE POOLED APH FARM YIELD DATA TO ESTIMATE THE REMAINING FARM LEVEL VARIABILITY AFTER ACCOUNTING FOR REGIONAL YIELD EVENTS

The next step in the process is to decompose the farm level variability in the APH pooled data set into two components—regional variability and the remaining or residual farm level variability after accounting for regional variability. This is accomplished by subtracting the CAR yield from the farm yield. This generates a deviation from the county yield ( $d_t^f$ ) for each farm  $f$  in the APH yield data set and for each year  $t$  of data reported. Farm  $f$ 's average deviation from the county ( $\bar{d}^f$ ) can then be calculated for each farm. These expressions can be presented mathematically as:

$$d_t^f = y_t^f - R_t^C \quad (5)$$

and

$$\bar{d}^f = \frac{1}{T_f} \sum_{t=1}^{T_f} d_t^f = \bar{y}^f - \bar{R}^C \quad (6)$$

After constructing  $d_t^f$  and  $\bar{d}^f$ , the remaining farm variability can be constructed as:

$$e_t^f = d_t^f - \bar{d}^f = (y_t^f - \bar{y}^f) - (R_t^C - \bar{R}^C) \quad (7)$$

Expression (7) indicates that the remaining farm residuals or variability  $e_t^f$  can be viewed as the difference of the farm's deviation from its average yield and the CAR's deviation from its average yield for the same period of time. Essentially each farm's deviation from its mean yield is adjusted by the amount by which the entire county deviated from its average. The total variation in farm yields has been decomposed into the variation that occurred at the regional level and the remaining farm level variation around the regional yields. The two sources of variation are reconstituted during the premia estimation process. Decomposing the variability in this manner allows the longer regional data set to be used to estimate the severity and frequency of large regional events.

If a given county has 50 or more farms with six (6) or more years of farm level data, the residuals from that county's farms alone are used in the rating process for that county. If a given county has less than 50 farms with six or more years of data the residuals from all farms in the risk rating region are used to develop the county's rates.

Before proceeding to the analysis of the price data, the residuals  $e_t^f$  are checked for heteroskedasticity with respect to  $R_t^C$ ,  $\bar{y}^f$ , or  $\bar{d}^f$ . If heteroskedasticity is indicated an appropriate GLS correction is made. The GLS correction has not generally been needed for the grain crops analyzed.

## ■ ESTIMATING THE PRICE-YIELD DISTRIBUTION

For traded crops, the futures board is used to obtain a forecast of fall prices. The relationship between price and yield historical deviations is estimated as:

$$(P_t^1 / P_t^0) = a_1^p + a_2^p \left( \frac{R_t^C}{\hat{R}_t^C} - \frac{1}{T_C} \sum_{t=1}^{T_C} \frac{R_t^C}{\hat{R}_t^C} \right) + e_t^p \quad (8)$$

where  $P_t^1$  is the harvest time price of the crop,  $P_t^0$  is the pre-planting time forecast of the harvest time price as indicated by the futures markets,  $R_t^C$  is the CAR yield, and  $\hat{R}_t^C$  is the forecasted CAR yield for year  $t$  as obtained from expression (4) i.e.  $\hat{R}_t^C = a_1^C + g(t)$ . The term inside the parentheses is constructed so as to generate a zero mean set of proportional regional yield deviations for the subset of the regional yield data used in expression (8). (There are usually not as many time series price observations as there are yield observations. As a result, the historical

$\frac{R_t^C}{\hat{R}_t^C}$  values may not exactly average to 1, that is, the long-term expected value of  $\frac{R_t^C}{\hat{R}_t^C}$  .)

In expression (8), price proportions are used as doing so allows the use of the historical prices without a need to deflate the price series. Historical real prices and real price deviations were examined and found to be heteroskedastic across time. Using price proportions was suggested by Jerry Skees, in early reviews of the IP product, and found to be fairly homoskedastic. Figure 5 plots historical price proportions for wheat from 1960 to 1996 (excluding 1973—see Appendix B). As can be seen it appears that the relative magnitudes of proportional price deviations has not changed across the years. Similar results were found for the other grain crops. Appendix B presents a more detailed discussion of the price data sources and additional diagnostics of the data sets.

## ■ ESTIMATING THE IP PREMIA

For a given farmer's data  $y_t$  for  $t=1,2,T_t$ , the farmer's premia can be estimated by the following process:

### ■ CALCULATE THE FARM'S APH YIELD

$$\bar{y} = \frac{1}{T_t} \sum_{t=1}^{T_t} y_t \quad (9)$$

### ■ CALCULATE THE AVERAGE CAR YIELD FOR THE SAME YEARS

$$\bar{R} = \frac{1}{T_t} \sum_{t=1}^{T_t} R_t^C \quad (10)$$

### ■ CALCULATE THE AVERAGE AMOUNT BY WHICH THE FARM DEVIATES FROM THE CAR YIELD

$$\bar{d} = \bar{y} - \bar{R} \quad (11)$$

As with the  $a_1^C$  values from expression (3a), the  $\bar{d}$  value from (11) is bootstrapped to account for the amount of uncertainty associated with the sample statistic (see Appendix C).

### ■ CALCULATE THE FARM'S POTENTIAL REVENUE INDEMNITY TRIGGERS

$$IT_i = i P_0 \bar{y} \quad \text{where } i = \% \text{ election} \quad (12)$$

The farmer electing to insure  $i$  of  $P_0 \bar{y}$  will receive an indemnity if per acre revenue falls below  $IT_i$ .

## ■ SIMULATE THE FARM'S REVENUE DISTRIBUTION

### ■ RANDOMLY DRAW

- $e_{t,97}^R$  from expression (2a)
- $e_t^f$  from expression (7)
- $e_t^p$  from expression (8)

### ■ CONSTRUCT A SIMULATED CAR YIELD AS :

$$R_s^C = a_1^C + g(1997) + e_{t,97}^R \quad (13)$$

### ■ CONSTRUCT SIMULATED FARM YIELD AS:

$$y_s = R_s^C + \bar{d} + e_t^f \quad (14)$$

### ■ CONSTRUCT SIMULATED PRICE REALIZATION AS:

$$P_1^s = P_0 (1 + a_2^p (\frac{R_s^C}{\hat{R}_s^C} - 1) + e_t^p) \quad (15)$$

### ■ CONSTRUCT A SIMULATED REVENUE REALIZATION AS:

$$REV_s = P_1^s y_s \quad (16)$$

If  $REV_s$  is less than  $IT_t$ , a “payment” or indemnity of  $(IT_t - REV_s)$  is assumed to be made and the amount recorded. The above simulation process is repeated 10,000 times and a running total of the “payoffs” is recorded for each of the possible indemnity levels. The average indemnity (the total indemnities divided by 10,000) is then used as an estimate of the actuarially neutral premia.

To account for statistical uncertainty in estimating the revenue distribution's tails, as well as in the effects of changing farm policies upon producer behavior, the estimated actuarially neutral premia are loaded by 20%. In addition, the 12% administrative load imposed by FCIC on their other insurance products is also added to the IP premia.

The above process can be used to determine a premium for any continuous combination of  $\bar{y}$  and  $\bar{R}$  values. To facilitate the desires of insurance companies, rates were developed for discrete combinations of farm and regional average yields. An example of the resulting tables is presented in Table2.

TABLE 1 --- FARM AND REGIONAL YIELD AND APH EXAMPLE

YEARS	COUNTY ADJUSTED REGIONAL YIELD	FARM ONE'S REPORTED YIELDS	FARM TWO'S REPORTED YIELDS
1996	30.6	30.6	30.6
1995	42.5	42.5	42.5
1994	33.5	33.5	33.5
1993	39.6	39.6	39.6
1992	21.1		21.1
1991	43.6		43.6
1990	36.2		36.2
1989	35.6		35.6
1988	14.6		14.6
1987	32.4		32.4
AVERAGE YIELD	32.97	36.55	32.97
CAR AVERAGE YIELD FOR CORRESPONDING YEARS		36.55	32.97
FARM'S AVERAGE DEVIATION FROM COUNTY YIELD		0	0
--TRENDED 1997 -- PROJECTED YIELD	35.5	35.5	35.5
PROJECTED REV (@\$4.00/BUSHEL)	\$142.00	\$142.00	\$142.00
75 % INDEMNITY TRIGGER		\$109.65	\$98.91
ACTUAL PERCENT COVERAGE		77%	70%



TABLE 2 -- EXAMPLE RATE FILE FOR CENTRAL MONTANA

```

=====
      year      state  county  crop   type  practice
      98         30      **    11    997   997
                election  percent  75
=====
                COUNTY INTERVALS
      CMIN         0    21    24    27    30    33    36    39    42
      CMAX         20   23    26    29    32    35    38    41   999
=====
FARM YIELD
INTERVALS
  0  15 ][ 75  119  164  222  291  378  480  591  796
 16  18 ][ 54  86  120  164  221  291  376  476  664
 19  21 ][ 45  72  101  139  188  251  327  417  594
 22  24 ][ 38  59  83  115  158  212  280  361  523
 25  27 ][ 38  50  71  99  135  183  243  317  465
 28  30 ][ 38  42  61  85  117  159  212  277  414
 31  33 ][ 38  38  52  74  102  139  185  244  368
 34  36 ][ 38  38  44  63  87  119  160  211  322
 37  39 ][ 38  38  38  55  76  105  141  187  287
 40  42 ][ 38  38  38  48  68  93  125  166  257
 43  45 ][ 38  38  38  43  60  83  112  149  230
 46  48 ][ 38  38  38  38  52  72  98  130  203
 49  51 ][ 38  38  38  38  47  65  88  118  183
 52  54 ][ 38  38  38  38  43  59  80  107  166
 55  57 ][ 38  38  38  38  39  54  73  97  151
 58  60 ][ 38  38  38  38  38  48  65  87  135
 61  63 ][ 38  38  38  38  38  45  60  80  124
 64  66 ][ 38  38  38  38  38  42  56  74  114
 67  69 ][ 38  38  38  38  38  39  52  69  106
 70  999 ][ 38  38  38  38  38  38  47  62  96
=====

```

# APPENDIX A – ESTIMATING YIELD TRENDS OVER TIME

Technological developments have increased crop yields over the past 50 years. To develop a distribution of expected crop yields from historical data, yield trends must be addressed. Various functional forms to model yield trends were evaluated. The functional forms chosen are flexible and approximate the usual expectation of yield trends. The functional forms that are statistically estimated using least squares procedures are:

$$R_t = a_1^R \quad (A1)$$

$$R_t = a_1^R + a_2^R t \quad (A2)$$

$$R_t = a_1^R + a_2^R t^{a_3^R} \quad (A3)$$

$$R_t = a_1^R + \frac{a_2^R t^2}{a_4^R + t^2} \quad (A4)$$

$$R_t = a_1^R + \frac{a_2^R t^{a_3^R}}{a_4^R + t^2} \quad (A5)$$

The choice of the appropriate functional form is made using standard F-tests.

## APPENDIX B: PRICE CHANGE DISTRIBUTIONS

### **Objective.**

Our goal was to develop accurate distributions for price changes in year  $t$  from planting ( $P_t^0$ ) until harvest period price ( $P_t^1$ ), given useable information at the planting period. These price distributions were then used for the revenue distribution ( $REV_s$ ). Useable information was defined as readily available before the sign-up date and easily verifiable. This distribution relied on monthly average time series data from well-established contracts traded on commodity futures exchanges. The prices from these futures contracts were found to closely correspond with commodity cash prices.

In addition to selecting the most appropriate futures contract, our analysis examined the effects of loan rates, stock levels, and estimates of pre-planting price variance on the price distribution. Again, all of these factors were defined so as to be readily available and verifiable at the sign-up period.

### **Data.**

Commodity Futures Exchanges. Monthly averages of the futures price contracts were constructed for use with each crop during the 27-year period from 1960 to 1996. The Chicago Board of Trade (CBOT) is the clear choice for corn and soybeans. Grain sorghum is not traded on a futures exchange, but our analysis showed cash grain sorghum prices to be closely matched by 90% of the cash corn price, so this was used as the grain sorghum price. A futures contract for cotton is traded only on the New York Cotton Exchange (NYCE), the contract used for the cotton price distribution.

Wheat futures contracts are traded on three exchanges — the CBOT, the Kansas City Board of Trade, and the Minneapolis Grain Exchange. Evaluation of the futures contracts

showed high correlation between monthly average prices on the nearby contract (the closest contract to the trading date) for the Chicago with both the Kansas City (.998) and the Minneapolis (.992) futures contracts. Since the Chicago contract has by far the largest trading volume, it was used instead of the other exchanges for wheat prices.

Dates. Planting period and harvest period dates were defined for each crop. The planting period price was always the average of a thirty day period ending two weeks before the crop insurance sign—up for that crop and location, while the harvest period price was an average 30 days the month prior to the close of the harvest futures contract (defined in Table B.1 below). The futures prices were from those compiled by Prophet Information Services.<sup>1</sup> The table below lists the commodity, contract, and the time periods used for the planting and harvest periods. There was one obvious outlier, 1973, that was not included in our price change distribution.

Table B.1

Commodity	Sign up Date	Futures Contract	Planting Period	Harvest Period
Corn	March 15	December corn, CBOT	February Average	November Average
Soybeans	March 15	November Soybeans, CBOT	February Average	October Average
Spring Wheat	March 15	September Wheat, CBOT	February Average	August Average
Winter Wheat (Montana and Washington)	September 30	Beginning: July Wheat, CBOT. Ending: September Wheat, CBOT	Average August 15 to September 14 on July	August Average on September
Winter Wheat (Kansas)	September 30	July Wheat, CBOT	Average August 15 to September 14 on July	June Average
Sorghum #1	January 15	September Corn, CBOT	90% December Average	90% of Average August 15 to September 14
Sorghum #2	February 15	September Corn, CBOT	90% January Average	90% of Average August 15 to September 14
Sorghum #3	March 15	December Corn, CBOT	90% of February Average	90% of November Average
Cotton	March 1	December Cotton, NYCE	Average from January 15 to February 14	November Average

<sup>1</sup> Futures Daily Data (1960-1996) CD-ROM. © Prophet Information Services, Palo Alto, CA.

As is clear from Table B.1, the dates used to define average prices for winter wheat and sorghum reflect these crops' multiple planting, sign—up, and harvest dates.

### **Adjustments Considered.**

There have been a number of important changes in USDA policy toward these crops that may have affected the variability of their prices in the past. In addition, there are a number of measures that have been hypothesized to influence price variability. We discuss the results of statistical tests for these effects below.

Commodity Loan Rates. We evaluated potential dampening effects of loan rates on price distributions for corn, wheat (spring and winter), soybeans, and cotton. The loan rate effects on the price changes were tested using a switching regression for all commodities above. Absolute price differences ( $apd = |P_t^1 - P_t^0|$ ) for those observations where the harvest price was less than the planting period price estimate were regressed on the switching point model through:

$$apd = [a_0 + a_1 * LD]d + [a_0 + a_1 * z] * (1 - d) \quad (B.1)$$

In (B.1), LD is the planting period price ( $P_t^0$ ) less the loan rate (all prices normalized for inflation), z is the switch point to be selected optimally, and d is an indicator taking the value 1 (0 otherwise) if LD is less than z. If the optimal z were outside the range of LD (or if only a small number of observations had  $d=0$  for the optimal z, there is little confidence in the estimate of z, the switch point. If z lies well in the interior of the observed LDs, then there is some confidence that the loan rates dampen the magnitude of the negative price changes.

We found no significant effects of the loan rate on these monthly average futures price movements through equation (B.1) for all commodities except for spring wheat. The spring wheat price change distribution was adjusted for the loan rate effects as estimated from equation (B.1).

Stocks. We hypothesized that higher levels of stocks might dampen the planting to harvest period price changes, particularly for price increases stemming from adverse crop growing conditions. To test this hypothesis, we ran two regression models. The first model regressed the absolute price difference for a commodity on stocks of that commodity. Stocks were lagged one year, consistent with the pre sign-up period information availability requirement. Another regression model tested if lagged stock levels affected positive and negative price differences differently. There was no significant affect of stock levels under either model formulation.

Estimated Pre-plant Variance. We further hypothesized that estimated (pre-plant) price variance might be useful information for range of the apd. One measure of the pre-plant variance was the sample variance for the price of the futures contract two weeks before the sign—up date. A regression was run with this pre-plant variance estimate as an explanatory variable with apd the dependent variable. No significant explanatory power was shown by this variance estimate for any commodity's price distribution.

Our analysis also evaluated the informational content of implied volatility's from futures options markets in a similar manner as that used for the pre-plant variance above, but found no statistical support for their use in developing price distributions.

### **The Distribution.**

After verifying that neither loan rates, stock levels, nor estimates of the pre-plant variance significantly affected the distribution, the 37 year series of planting and harvest period prices were used to construct a price change distribution. This distribution was combined with the yield distribution described above to create a revenue distribution.

The price change distributions were defined as the ratio of the harvest period price to the planting period price as  $P_t^1/P_t^0$ .

Graphs of these price ratios are given below for corn, cotton, soybeans, and winter wheat in Kansas. Graphs of the other commodities are much the same.

## APPENDIX C: BOOTSTRAPPING PROCEDURES

In the IP rating process, bootstrapping is used to approximate the uncertainty associated with using estimated parameter levels to develop premia rates. To clarify the previous discussion the details of the bootstrapping procedures were not included in the main body of the report. This appendix demonstrates how bootstrapping is used to account for the degree of uncertainty associated with the estimates of  $a_1^C$  (from expression (3a)) and  $\bar{d}$  (from expression (11)).

Bootstrapping is essentially a monte carlo exercise which uses computer power to approximate the sampling characteristics of sample statistics. Similar procedures have long been discussed in the literature but the increased availability and reduced cost of computing power has made these procedures more widely useful in recent years. While bootstrapping is not likely to replace mathematical theory, it has proven useful in numerous applications. The reader is referred to the list of publications at the end of this appendix. The chapter by Jeong and Maddala as well as the monograph by Efron and Tibshirani is recommended.

■ To bootstrap the  $a_1^C$  values proceed as follows:

1. Randomly draw  $T_R$  (with replacement)  $e_{t,97}^R$  values from expression (2a).
2. Construct  $T_R$  CAR bootstrap realizations as :

$$R_{t,j}^C = a_1^C + g(1997) + e_{t,97}^R \quad (C-1)$$

In expression (C-1), the bootstrapped realizations have been scaled to 1997 units since the errors have been scaled to 1997 units in expression (2a).

3. Compute the j'th bootstrapped value for  $a_1^C$  i.e.

$$a_{1,j}^C = \frac{1}{T^R} \sum R_{t,j}^C \quad (C-2)$$



■ To bootstrap the  $\bar{d}$  values proceed as follows:

1. Randomly draw  $T_i$  (with replacement)  $e_t^f$  values from expression (7).
2. Construct  $T_i$  bootstrapped  $d_{t,j}$  realizations as:

$$d_{t,j} = \bar{d} + e_t^f \quad (C-3)$$

3. Compute the  $j$ 'th bootstrapped value for  $\bar{d}$  i.e.

$$\bar{d}_j = \frac{1}{T^j} \sum d_{t,j} \quad (C-4)$$

The above steps are repeated a large number of times (J) until there are J bootstrapped estimates of  $a_{1,j}^C$  and  $\bar{d}_j$ . The premia estimation procedure then proceeds.

## PREMIA ESTIMATION WITH BOOTSTRAPPING INCLUDED

With bootstrapping included the premia estimation procedure is slightly modified from the procedure presented in the main body of the text. The corresponding equations have been reproduced below.

### ■ RANDOMLY DRAW

- $e_{t,97}^R$  from expression (2a),  $e_t^f$  from expression (7),  $e_t^p$  from expression (8), an  $a_{1,j}^C$  value from expression (C-2), and a  $\bar{d}_j$  value from expression (C-4).

■ CONSTRUCT SIMULATED CAR YIELDS AS :

$$R_s^C = a_1^C + g(1997) + e_{t,97}^R \quad (13)$$

$$R_{s,j}^C = a_{1,j}^C + g(1997) + e_{t,97}^R \quad (13a)$$

■ CONSTRUCT SIMULATED FARM YIELD AS:

$$y = R_s^C + \bar{d} + e_t^f \quad (14a)$$

$$y_j = R_{s,j}^C + \bar{d}_j + e_t^f \quad (14b)$$

$$y_s = y + (y + y_j) \quad (14)$$

■ CONSTRUCT SIMULATED PRICE REALIZATION AS:

$$P_1^s = P_0 (1 + a_2^p (\frac{R_s^C}{\hat{R}_s^C} - 1) + e_t^p) \quad (15)$$

■ CONSTRUCT A SIMULATED REVENUE REALIZATION AS:

$$REV_s = P_1^s y_s \quad (16)$$

If  $REV_s$  is less than  $IT_t$ , a “payment” or indemnity of  $(IT_t - REV_s)$  is assumed to be made and the amount recorded. The above simulation process is repeated 10,000 times and a running total of the “payoffs” is recorded for each of the possible indemnity levels. The average indemnity (the total indemnities divided by 10,000) is then used as an estimate of the actuarially neutral premia.

As indicated in the main body of the report, the estimated actuarially neutral premia is loaded by 20%. In addition, the 12% administrative load imposed by FCIC on their other insurance products is also added to the IP premia.

#### SUGGESTED REFERENCES

Bernard, J. and M. R. Veall. "The Probability Distribution of Future Demand." JOURNAL OF BUSINESS AND ECONOMIC STATISTICS. 5(July,1987):417:423

Efron, B. and R. J. Tibshirani. AN INTRODUCTION TO THE BOOTSTRAP. Chapman and Hall, New York, 1993.

Jinook, J. and G. S. Maddala. "A Perspective on Application of Bootstrap Methods in Econometrics." In Maddala, Rao, and Vinod, Editors of HANDBOOK OF STATISTICS, Vol. 2. North Holland, Amsterdam, (1993):573-610.

Prescott, D. M. and T. Stengos. "Bootstrapping Confidence Intervals: An Application to Forecasting the Supply of Pork." AMERICAN JOURNAL OF AGRICULTURAL ECONOMICS. 69(1987):266-273.



FIGURE 2 -- MONTANA REGIONAL WHEAT YIELDS EXAMPLE

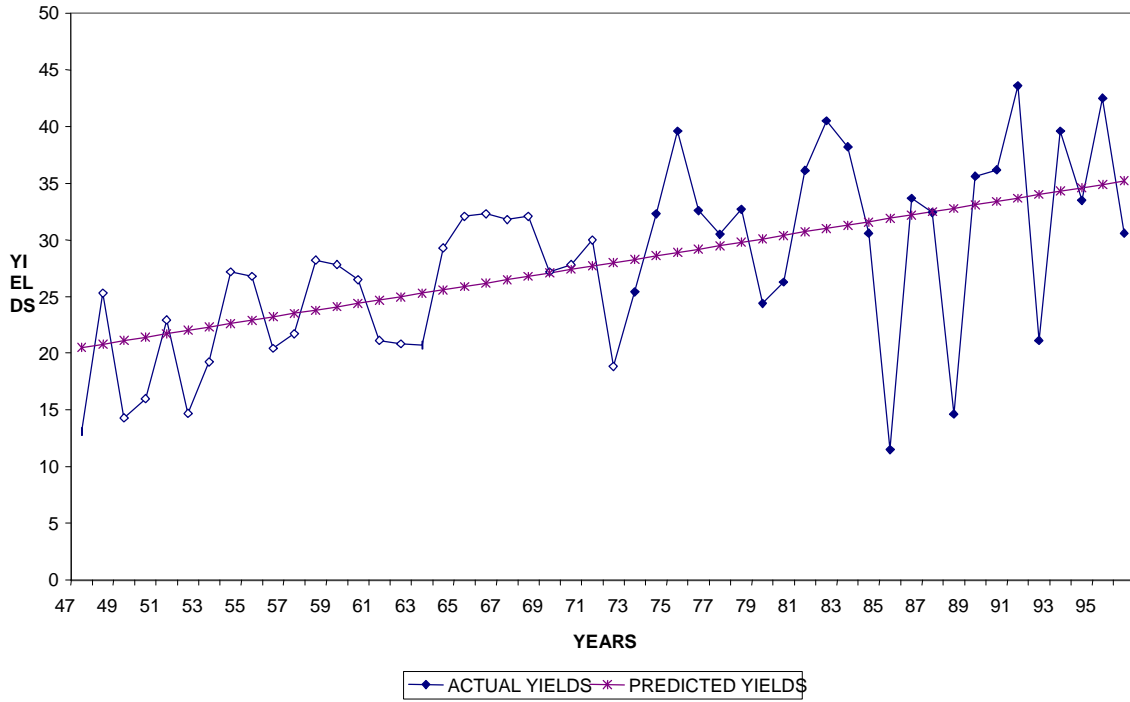
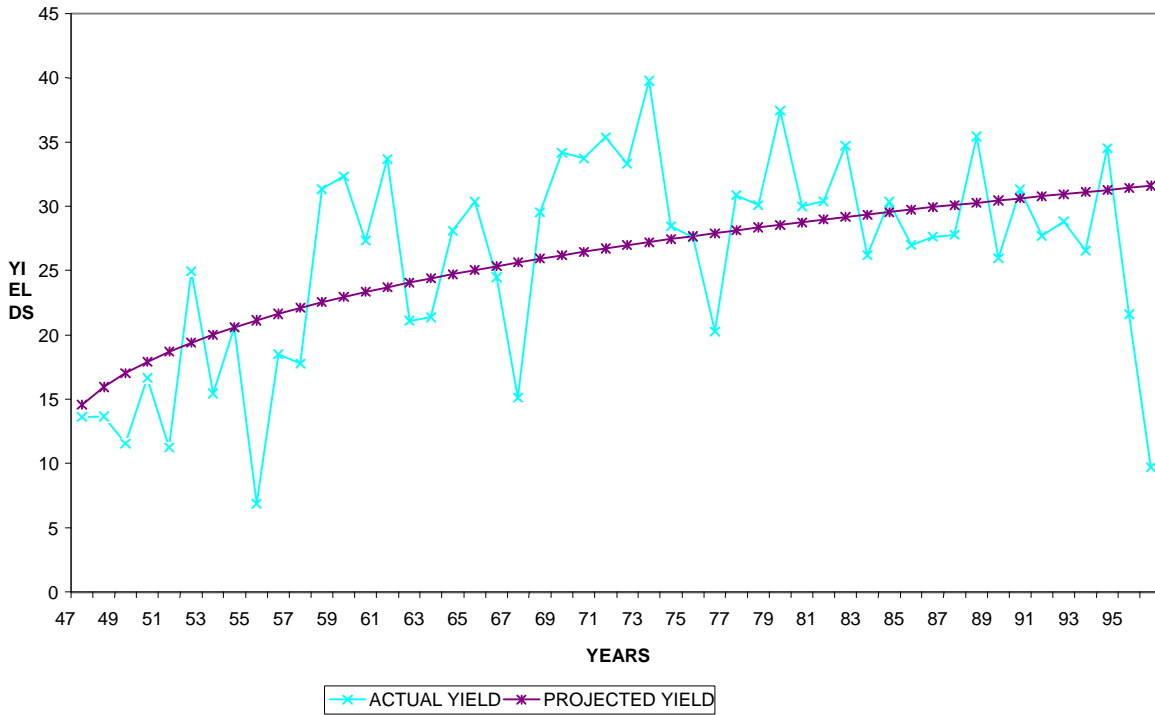
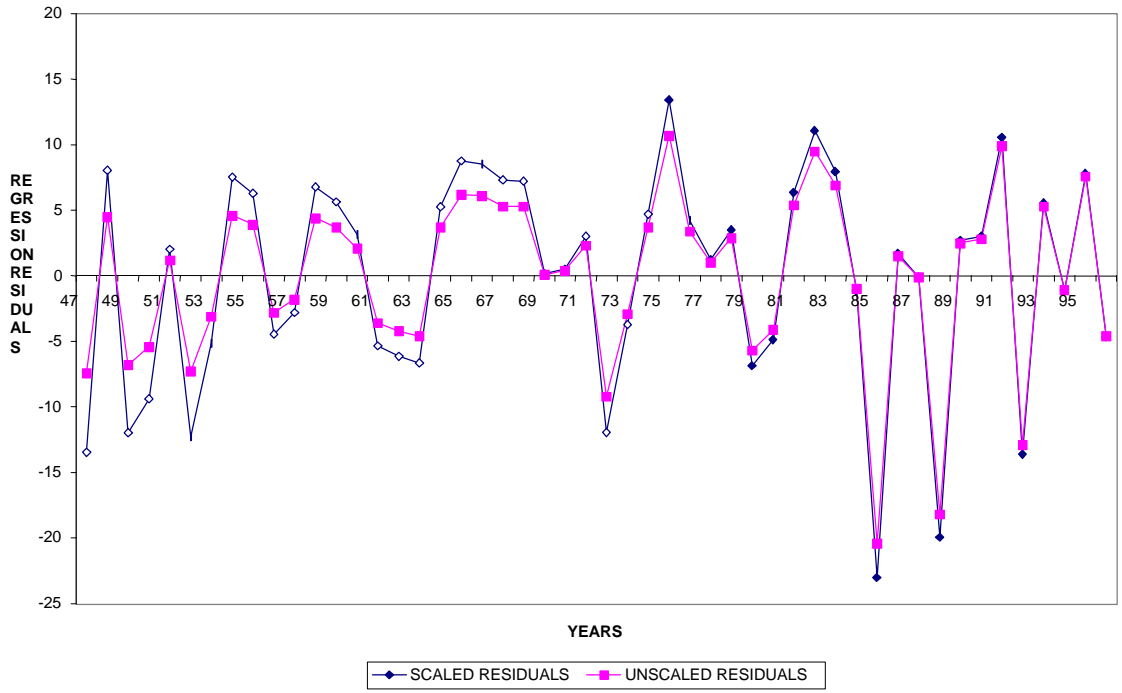


FIGURE 3 -- KANSAS REGIONAL WHEAT YIELDS EXAMPLE



**FIGURE 4 -- UNSCALED VS. SCALED REGIONAL WHEAT RESIDUALS**



**FIGURE 5 -- PROPORTIONAL WHEAT PRICES**

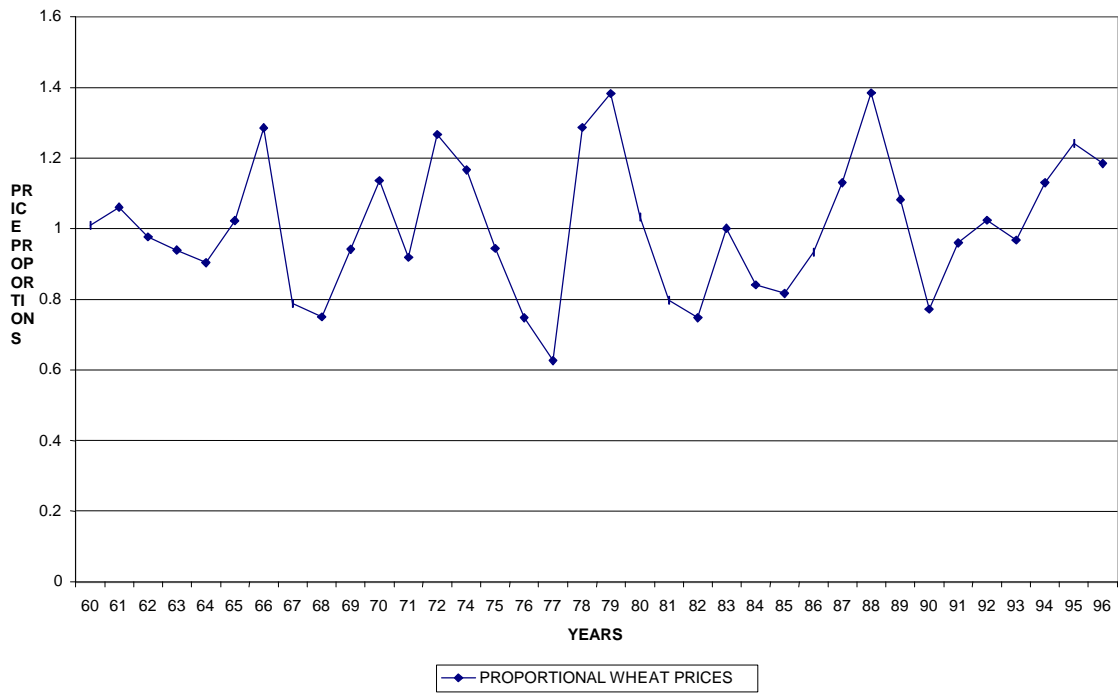


FIGURE B1 -- CORN HARVEST PRICE/PLANTING PRICE

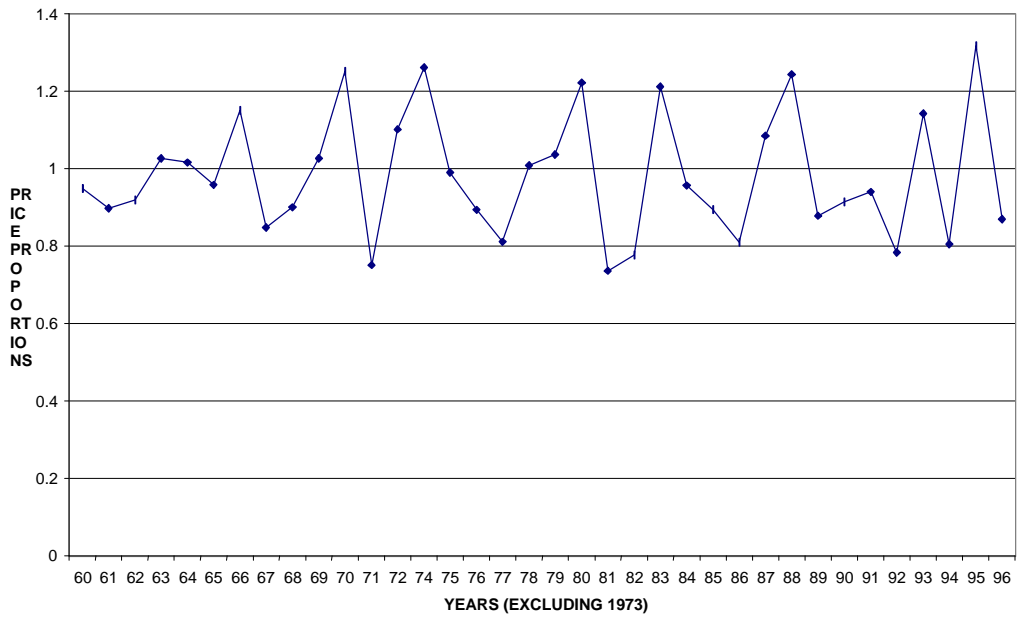


FIGURE B2 -- COTTON HARVEST PRICE/PLANTING PRICE

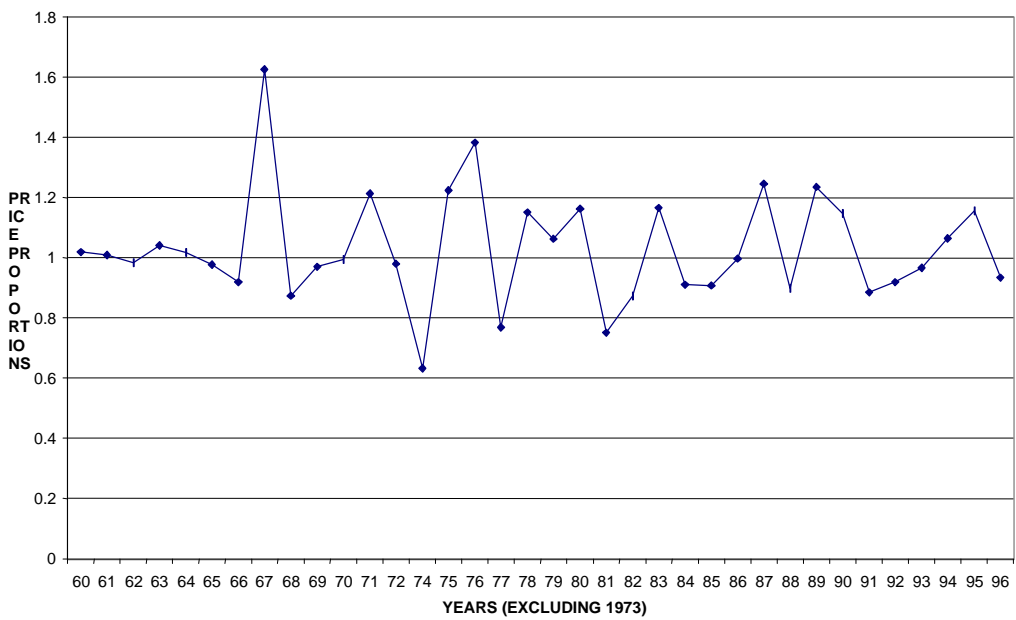


FIGURE B3 -- KANSAS WINTER WHEAT HARVEST PRICE/PLANTING PRICE

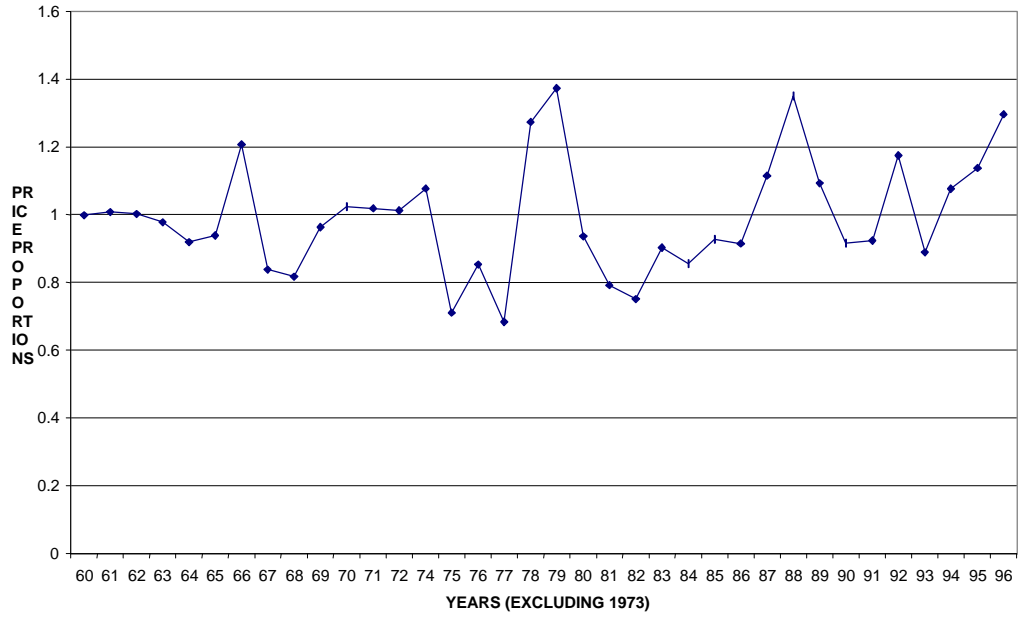


FIGURE B4 -- SOYBEAN HARVEST PRICE/PLANTING PRICE

